

Controlling spontaneous-emission noise in measurement-based feedback cooling of a Bose-Einstein Condensate

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A Bose-Einstein condensate (BEC) undergoing measurement-based feedback cooling is analysed with a full quantum-field simulation. Two experimental setups are considered: a BEC in a cavity and a trapped BEC undergoing phase-contrast imaging. An experimentally important parameter regime is found where the simulation results diverge from the predictions of both single-mode quantum models and multi-mode semiclassical models. In this regime, simple cooling schemes cannot control the quantum noise limited heating due to measurement backaction. We describe a feedback scheme that is specifically adapted to the measurement, which can still produce cooling.

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A Bose-Einstein condensate (BEC) in a dilute atomic gas is the premier platform for the investigation of quantum fields. Since it can be well isolated from its environment and is highly controllable using a combination of optical, rf and magnetic fields [1], it has the potential to address a broad range of research questions in fundamental and applied science. These include studies in quantum nonequilibrium thermodynamics [2], entanglement of massive particles [3] and the quantum simulation of both cosmological phenomena, such as Hawking radiation emitted from a black hole's event horizon [4], and phase transitions in condensed matter, including superconductivity and quantum magnetism [5]. Furthermore, Bose-condensed sources are likely to be key components in a range of future technologies, such as improved precision inertial sensors based on atom interferometry, where the sensitivities of current devices are limited by the properties of thermal sources [6, 7]. This wealth of research opportunity has therefore led to great practical interest in the ability to control the spatial state of the BEC's quantum field. Research has predominantly focussed on open-loop control of the condensate by direct control of optical and magnetic potentials. In contrast, continuous measurement feedback control of BECs is rarely employed due to the perceived fragility of the quantum state to measurement backaction. However, this intuition does not account for the advantages active feedback control can provide. Active feedback can control properties of the BEC using the information gained from the continuous measurement, which includes properties that would have been adversely affected by the measurement backaction. Furthermore, active feedback provides robust and reliable behaviour that can not be matched by open-loop control schemes [8]. The advantages of feedback controlling a BEC have been demonstrated for a single-mode model [9, 10] and

for a multi-mode BEC under the semiclassical approximation [11, 12]. However, an investigation of a feedback-controlled Bose gas that incorporates both the full quantum field and the multi-mode dynamics has not been considered. In this letter we show that a correctly designed feedback-control scheme can indeed cool a degenerate Bose gas to a steady state. Importantly, we show that this feedback scheme *must* be designed with a full quantum-field simulation in order to account for the effects of spontaneous emission.

Continuous measurement of BEC has been achieved using optical fields, but heating due to spontaneous scattering of the photons placed strong limits on the lifetime of the condensate [13, 14]. The fundamental limit to this heating can only be improved by placing a BEC in a cavity [15–17], but it is unknown whether using the measurement information to perform active feedback can overcome this heating and cool a condensate. This problem was investigated using Gaussian and semiclassical approximations in [11, 12, 18]. Under the Hartree-Fock semiclassical method, which assumes the condensate has a fixed number of atoms, all the same single-particle state, it appeared that it was possible to cool a BEC under the effect of a measurement. However, without a full-field calculation it is difficult to determine if the heating process has been properly characterised. Full quantum-field calculations of BECs undergoing continuous measurement were made possible by a recent tool based on the Number-Phase Wigner (NPW) phase-space distribution [19–21], which allows the simulation of a cooled BEC with realistic measurement strengths.

We examine both a BEC in a cavity and a BEC undergoing density measurement via phase-contrast imaging. In both cases, for sufficiently weak measurement the Hartree-Fock simulation matches the full-field simulation and the BEC can be cooled using feedback. How-

ever, under more typical experimental parameters, the Hartree-Fock simulation fails to predict a heating effect caused by the scattering process in the measurement. For both systems new controls are proposed, and the full-field NPW simulation demonstrates that net cooling can still be achieved during high resolution measurement.

Cavity-mediated Measurement: We consider cooling a BEC with a measurement mediated by a cavity, as shown in Fig. 1. This feedback mechanism has been studied extensively for neutral atoms and optomechanical resonators [22–24]. Furthermore this measurement has been demonstrated experimentally for optomechanical resonators [25] and ultracold atoms [26, 27]. Light is passed through an optical cavity containing the trapped BEC. When the light is sufficiently detuned from the atoms such that the excited level is minimally populated, and the cavity is sufficiently lossy that the cavity dynamics can be adiabatically eliminated, then the phase of the transmitted light contains a signal proportional to the overlap of the atomic density with the spatial envelope of the optical field in the cavity. A homodyne measurement of this phase can then be used to continually update the best estimate of the atomic state, known as the *filter*. The mean position of the trap can then be moved by an amount proportional to the negative of the BEC's bulk momentum, which will cool the BEC. The equation of motion for the filter that governs the single atom version of this system is found in [22, 28]. Generalising this filter evolution to apply to a multi-atom quantum field with a negligible inter-atomic interactions and converting it to Stratonovich form gives:

$$\partial_t \hat{\rho} = -i[\hat{H}_F(u_s), \hat{\rho}] + \gamma \mathcal{D}[\hat{C}_\xi] \hat{\rho} + \gamma \mathcal{C}[\hat{C}_\xi] \hat{\rho} + \sqrt{\gamma} \mathcal{H}[\hat{C}_\xi] \hat{\rho} \eta, \quad (1)$$

where energy is in units of $\hbar\omega$, position has units of $\sqrt{\hbar/(m\omega)}$ and time has units of $1/\omega$; $\hat{\rho}$ is the conditional state of the atomic field; γ is the measurement strength; $\mathcal{D}[\hat{c}] \hat{\rho} \equiv \hat{c} \hat{\rho} \hat{c}^\dagger - \frac{1}{2}(\hat{c}^\dagger \hat{c} \hat{\rho} + \hat{\rho} \hat{c}^\dagger \hat{c})$ is the decoherence superoperator; $\mathcal{H}[\hat{c}] \hat{\rho} \equiv \hat{c} \hat{\rho} + \hat{\rho} \hat{c}^\dagger - \langle \hat{c} + \hat{c}^\dagger \rangle \hat{\rho}$ is the innovations superoperator; $\mathcal{C}[\hat{c}] \hat{\rho} \equiv \langle \hat{c} + \hat{c}^\dagger \rangle \mathcal{H}[\hat{c}] \hat{\rho} - \frac{1}{2} \mathcal{H}[\hat{c}^2] \hat{\rho} + \langle \hat{c}^\dagger \hat{c} \rangle \hat{\rho} - \hat{c} \hat{\rho} \hat{c}^\dagger$ is the Stratonovich correction superoperator, and η is a Stratonovich differential that is derived from the measurement signal in the laboratory. $\hat{C}_\xi = \int dx \hat{\psi}^\dagger(x) c_\xi(x) \hat{\psi}(x)$ is the measurement operator where $c_\xi(x) = \cos^2(\xi x - \frac{\pi}{4})$ defines the intensity of the intracavity optical field. The atomic-field Hamiltonian $\hat{H}_F = \int dx \hat{\psi}^\dagger(x) h_F(x, u_s) \hat{\psi}(x)$ depends on $h_F(x, u_s) = \frac{1}{2}(-\partial_x^2 + x^2) + u_s x \langle \hat{P} \rangle / \langle \hat{N} \rangle$, the single-particle Hamiltonian containing the BEC's kinetic energy, potential energy due to harmonic trap, and feedback. $\hat{P} = \int dx \hat{\psi}^\dagger(x) (-i\hat{\psi}'(x))$ is the full-field momentum operator where $\hat{\psi}'(x) = \partial_x \hat{\psi}(x)$, and $\hat{N} = \int dx \hat{\psi}^\dagger(x) \hat{\psi}(x)$ is the multi-particle number operator. $\xi = 2\pi x_0/\lambda$ is the ratio of the natural length scale of the trap x_0 and the wavelength λ of the optical cavity [22], and ω is the angular

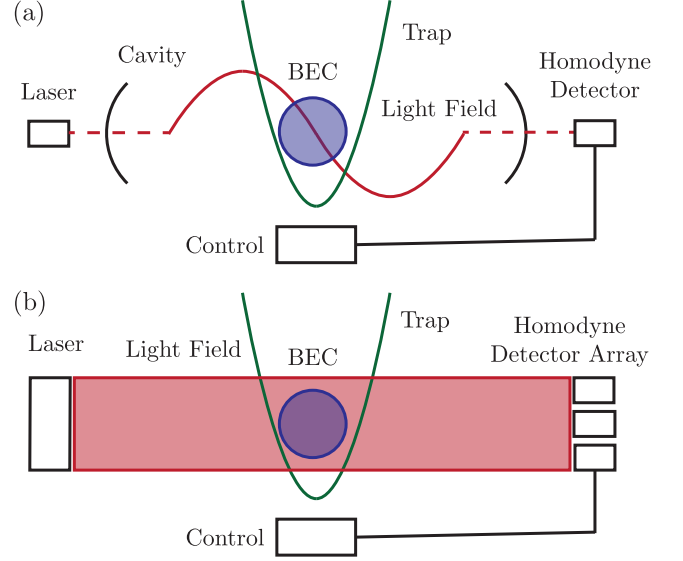


FIG. 1: Schematic diagram of a BEC under control. In (a) the measurement is mediated by a cavity as the homodyne signal is proportional to a density moment of the atomic cloud. In (b) a direct image of the BEC is taken using phase-contrast imaging. In both the controls are applied to the magnetic field with the aim of taking the BEC as close as possible to the ground state.

frequency of the harmonic trap.

In the limit $\xi \ll 1$, the cosine-squared term is approximately a position measurement. A BEC under a position measurement was used as a testing ground for the NPW simulation method due to the existence of reliable benchmarks [21]. However in contemporary experiments a very small ξ can be difficult to achieve so we will consider the more moderate values of $\xi = 0.1$ and $\xi = 0.5$. We will complete simulations using both the Hartree-Fock and NPW method. While the NPW method encapsulates a more complete description of the physics, the Hartree-Fock simulation is faster to perform as it assumes that the atoms occupy the same single-particle state. This state defines a macroscopic ‘wavefunction’ $\alpha_H(x)$ for the atoms, which obeys the following evolution equation:

$$\partial_t \alpha_H(x) = (-i h_F(x) - \gamma (c_\xi(x))^2 - 2c_\xi(x) C_\xi) \alpha_H(x) + \sqrt{\gamma} C_\xi \eta \alpha_H(x), \quad (2)$$

where $C_\xi = \int dx c_\xi(x) |\alpha_H(x)|^2$.

The NPW method provides a full-field solution to the master equation given in Eq. (1) [21]. The solution contains a function of the same dimension as the Hartree-Fock order parameter, α_N , but physical predictions are only obtained by averaging over multiple realisations of an extra noise increment η_f . Integrated alongside the fields α_N are a set of complex-valued variables w , and the lowest order expectation values of operators are extracted by the following weighted averages of these func-

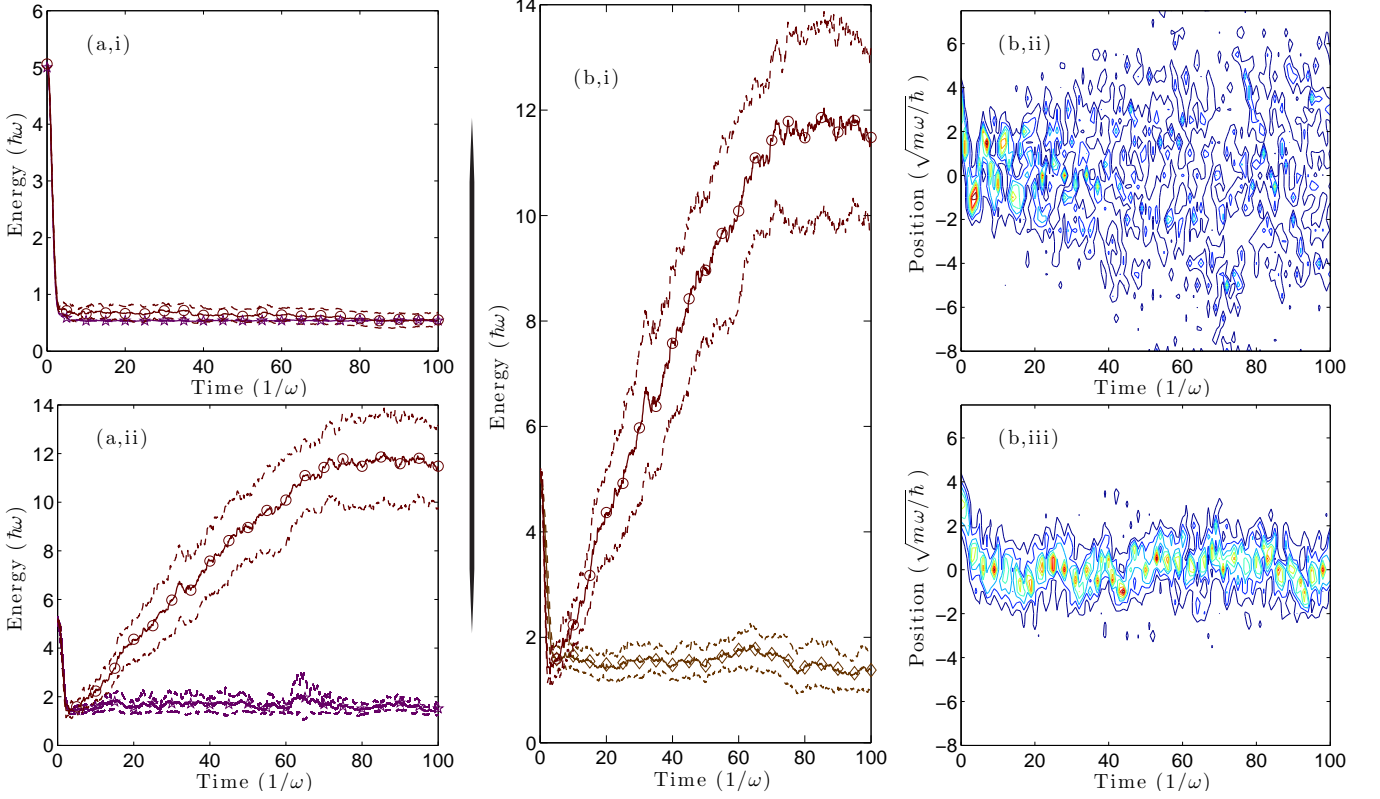


FIG. 2: (a) Comparison of the Hartree-Fock method versus NPW for simulation of a cooled BEC in a cavity for different ξ strengths. The NPW representation was integrated using Eq. (4), averaged over 10 paths, and plotted in red with circles. The Hartree-Fock representation was integrated using Eq. (2), averaged over 100 paths, and plotted in purple with stars. A small value of $\xi = 0.1$ is plotted in (a,i) while a higher strength of $\xi = 0.5$ is plotted in (a,ii). Both plots have $\gamma = 5$, $N = 100$ and $u_s = 1$. The Hartree-Fock method only agrees with NPW when $\xi = 0.1$, it fails to predict the additional heating present in the system when $\xi = 0.5$. (b) A comparison of a controlled BEC under direct measurement without and with the quantum noise control averaged over 10 paths. (b,i) shows a comparison of the per particle energy of the system, while the density fluctuations in the system without and with control are shown in (b,ii) and (b,iii) respectively. In (b,i): integration of the NPW representation Eq. (4) without quantum noise control, $u_c = 0$, is plotted with red with circles; integration with the addition of the quantum noise control, $u_c = 5$, is plotted in orange with diamonds. (b,ii) and (b,iii) are contour plots of the BEC's density, all lines indicate equidensity parts of the BEC, the colour range of the lines range from (light) blue hues to (dark) red hues to indicate low density to high density respectively. All integrations were performed with $u_s = 1$, $\xi = 0.5$, $\gamma = 5$ and $N = 100$.

tions [20]:

$$\langle \hat{a}_i^{(\dagger)} \rangle = \mathbb{A}[\alpha_i^{(*)}] \quad \langle : \hat{a}_i \hat{a}_j^\dagger :_{sym} \rangle = \mathbb{A}[\alpha_i \alpha_j^*] \quad (3)$$

where $::_{sym}$ denotes symmetric ordering and $\mathbb{A}[g(\alpha)] = \sum_n w_n g(\alpha_n) / \sum_n w_n$ is the weighted average over the different stochastic realisations of the noises η_f . The weighted stochastic differential equations (WSDEs) for a BEC in a cavity are [21]:

$$\begin{aligned} \partial_t \alpha_N(x) &= -i(h_F(x, u_s) + \sqrt{\gamma} c_\xi(x) \eta_f) \alpha_N(x), \\ \partial_t w &= (-2\gamma(C_\xi^2 - 2C_\xi \mathbb{A}[C_\xi]) + 2\sqrt{\gamma} C_\xi \eta) w. \end{aligned} \quad (4)$$

where $C_\xi = \int dx c_\xi(x) |\alpha_N(x)|^2$.

The comparison of the Hartree-Fock and NPW simulations [41] of a BEC in a cavity are shown in Fig. 2-(a),

where the initial state is a coherent state with a spatial amplitude given by a displaced Gaussian $\phi(x) = (2\pi)^{-1/4} \sqrt{N/\sigma} \exp(-(x - x_0)^2/4\sigma^2)$ where x_0 is the gaussian's displacement from the origin, σ is its variance and N is the average total number of the condensate. When the cosine measurement is close to a position measurement, $\xi = 0.1$, we see the NPW and Hartree-Fock simulations match well, with both predicting cooling of the BEC. For larger ξ the cosine measurement is no longer 'position-like', in that it has some curved structure on the scale of the BEC. When $\xi = 0.5$ we see the full-field solution predicts significantly more heating than the Hartree-Fock solution. If an experiment relied on the Hartree-Fock solution they would see unexpected *irremovable* extra heating in their system, demonstrating the importance of having these full quantum-field solu-

tions when examining quantum control of a BEC.

The Hartree-Fock method misses the additional heating in the BEC because it neglects certain spontaneous emission events. The measurement performed on the BEC is derived by assuming the atoms that constitute the BEC can be described as a two-level system interacting with off-resonant light. The light is sufficiently off-resonant that we can adiabatically eliminate the excited state. All the spontaneous emission events in the system are now encoded as scattering events between the light and BEC. These spontaneous emission events change the phase of the outgoing probe beam, which in turn is how the cosine-squared moment of the BEC is measured. As the Hartree-Fock approximation assumes all atoms are in the same single-particle state, it restricts which spontaneous emission events can be described. In contrast, the NPW method also allows non-collective states of the atoms, and can thereby simulate *all* spontaneous emission events. We will now demonstrate that this new simulation method can be used to design and test a control scheme that can cool the BEC even in the presence of this spontaneous emission noise.

The fact that the Hartree-Fock simulation worked when $\xi = 0.1$ shows that the net effect of the heating due to the spontaneous emission is partially mode-dependent. The net effect of the feedback results from the competition between the heating due to the scattering and the cooling due to the feedback. The feedback in the simulation was cooling the centre-of-mass motion of the BEC, which for $\xi = 0.1$ is very close to the spatial mode of the heating due to measurement backaction. When $\xi = 0.5$, energy was added to modes *other* than the position moment mode, so we hypothesise that the solution is to *add an extra control* which targets the *cosine-squared mode* of the BEC.

A methodology used to damp specific modes of a BEC is described in [29]. We can apply this method to create a control that targets the cosine-squared, $c_\xi(x)$, mode of the BEC. The new single-particle Hamiltonian that includes this new *quantum noise control* is

$$h_{FC}(x, u_s, u_c) = h_F(x, u_s) + \frac{u_c c_\xi(x)}{\langle \hat{N} \rangle} \times \int dy c'_\xi(y) \text{Im}[\langle \hat{\psi}^\dagger(y) \hat{\psi}'(y) \rangle], \quad (5)$$

where u_c is the strength of the quantum noise control for the cavity-mediated measurement and $c'_\xi(x) = \partial_x c_\xi(x)$.

NPW simulations of BEC cooling with this new control were performed without and with the novel quantum noise control, and the results are shown in Fig. 2-(b). In part (b,i) we can see the heating due to quantum noise is completely cancelled by the quantum-noise control. Comparing parts (b,ii) and (b,iii) we see that the quantum noise control vastly improves the stability of the BEC's density. In (b,ii) we see the quantum noise causes the BEC to break apart. In contrast, (b,iii) shows that the quantum noise control stops this spread.

The conditional master Eq. (1) that describes the cavity-mediated measurement is only valid when inter-atomic interactions are negligible. Although a non-interacting condensate can be created with a dilute atomic sample or via a Feshbach resonance [1], it is technically demanding. Furthermore, there are instances where large inter-atomic interactions are desirable, such as for stable atom laser operation [30, 31] and for the generation of nonclassical atom laser states [32]. Fortunately, feedback-control is still possible in this regime under a direct measurement of the BEC via off-resonant light.

Direct Measurement : Real-time continuous measurement of a BEC has already been achieved in the laboratory using phase-contrast imaging [13, 33], where an off-resonant laser beam passes through the BEC, and the resulting phase shift is proportional to the atomic column density. The conditional master equation for a quasi-one-dimensional ‘cigar-shaped’ BEC under phase-contrast imaging is derived in [11], and in Stratonovich form is:

$$d\hat{\rho} = -i[\hat{H}_{FN}(u_s), \hat{\rho}] + \gamma \int dx \mathcal{D}[\hat{M}_\nu(x)] \hat{\rho} + \gamma \int dx \mathcal{C}[\hat{M}_\nu(x)] \hat{\rho} + \sqrt{\gamma} \int dx \mathcal{H}[\hat{M}_\nu(x)] \hat{\rho} \eta(x), \quad (6)$$

where $\hat{H}_{FN} = \hat{H}_F + \frac{U}{2} \int dx \hat{\psi}^\dagger(x)^2 \hat{\psi}(x)^2$ contains the atomic nonlinearities, which can dramatically modify the behaviour of a trapped BEC. $\hat{M}_\nu(x) = (\mu_\nu * \hat{\psi}^\dagger \hat{\psi})(x)$ is

the measurement operator, which is the density operator convolved with a kernel function $\mu_\nu(x)$ where we use $*$ to indicate a convolution defined as $(f * g)(x) =$

$\int_{-\infty}^{\infty} dy f(y) g(x-y)$. This kernel function depends on the resolution length scale $nu = x_{\perp}/k_0 x_0^2$, where x_{\perp} is the size of the condensate in the tight trapping directions, $x_0 = \sqrt{\hbar/m\omega}$ for loose trapping frequency ω , and $k_0 = 2\pi/\lambda$ is the wavenumber of the probe light. Typically, $nu \ll 1$. The kernel function in Fourier space is given by $\mu_{\nu}(k) = \exp(-\nu k^4)$.

In this system, the measurement signal provides a resolution-limited density measurement of the BEC. Unlike the cavity-mediated measurement, in no limit does the signal provide a position measurement. We investigate two values for ν : $\nu = 10$ and $\nu = 0.1$, which provide ‘coarse’ and ‘fine’ measurements of the BEC density, respectively.

We examine the evolution of Eq. (6) after application of the Hartree-Fock method [12]:

$$\begin{aligned} \partial_t \alpha_H(x) = & \left(-i(h_F(x, u_s) + UN|\alpha_H(x)|^2) \right. \\ & + 2\gamma(\mu_{\nu} * \mu_{\nu} * |\alpha_H|^2)(x) \\ & \left. + \sqrt{\gamma}(\mu_{\nu} * \eta)(x) \right) \alpha_H(x), \end{aligned} \quad (7)$$

where the Hartree-Fock order parameter $\alpha_H(x)$ is normalised to 1, so we must include a factor of N in the non-linearity term, accounting for the N atoms of the condensate. The NPW method converts Eq. (6) to [19, 21, 34]:

$$\begin{aligned} \partial_t \alpha_N(x) = & -i(h_F(x, u_s) + U|\alpha_N(x)|^2) \\ & + \sqrt{\gamma}(\mu_{\nu} * \eta^f)(x) \alpha_N(x), \\ \partial_t w = & \int dx (-2\gamma((M_{\nu}(x))^2 - 2M_{\nu}(x)\mathbb{A}[M_{\nu}(x)]) \\ & + 2\sqrt{\gamma}M_{\nu}(x)\eta(x))w, \end{aligned} \quad (8)$$

where $M_{\nu}(x) = (\mu_{\nu} * |\alpha_N|^2)(x)$.

A comparison of the Hartree-Fock method to the NPW method for different strengths of ν is shown in Fig. 3-(a). This analysis is performed *without* a quantum noise control. When $\nu = 10$ we see the Hartree-Fock method agrees with NPW, and both predict BEC cooling. This shows that for low resolution density measurements, higher-order modes are weakly excited such that feedback to the position mode provides sufficient damping, and the energy of the BEC stays low. Unfortunately, reaching this parameter limit is difficult with current traps [12]. When the BEC is probed at a finer spatial resolution ($\nu = 0.1$), the Hartree-Fock solution *diverges* from the NPW solution, which shows that the quantum noise of the spontaneous scattering events excites higher-order modes of the BEC that are not cooled by the position control. This results in net heating. As with the cavity-mediated measurement, we aim to introduce controls for higher-order modes to compensate for the heating of the BEC. For this multi-channel measurement we shall require multiple channels of feedback, so

we introduce an arbitrary time-varying potential for the atomic field. There are several proposals for implementing such potentials experimentally [35–39]. The single-particle Hamiltonian $h_{FM}(x, u_s, u_{\mu})$ describing this new feedback is:

$$\begin{aligned} h_{FM}(x, u_s, u_{\mu}) = & h_F(x, u_s) + \frac{u_{\mu}}{\langle \hat{N} \rangle} \\ & \times (\mu_{\nu} * \mu'_{\nu} * \text{Im}[\langle \hat{\psi}^{\dagger} \hat{\psi}' \rangle])(x), \end{aligned} \quad (9)$$

where u_{μ} is the strength of the quantum noise control for the direct measurement.

The novel quantum noise control defined in Eq. (9) is simulated in Fig. 3-(b). In part (b,i) we can see the novel control completely cancels the heating caused by the strong direct measurement of the BEC. The heating produced in a continuum of modes is efficiently damped by the field of controls. In parts (b,ii) and (b,iii) we compare the waveform of the BEC without and with the quantum noise control respectively. In (b,ii) we see the quantum noise causes the BEC to spread rapidly. In contrast, (b,iii) shows that the quantum noise control stops this distortion. The bulk of the BEC remains close to the centre of the trap.

In both the direct measurement and the cavity-mediated measurement we have shown cooling of a condensate is possible for a range of parameters with an appropriate choice of control. Experimentally, it is tempting to try and cool a BEC in the ‘weak’ probing limit ($\xi = 0.1$ for the cavity mediated and $\nu = 10$ for phase-contrast). In this case only the bulk feedback mechanism, governed by parameter u_s and defined in Eq. (1), is required to achieve net cooling. However, operating in this parameter regime requires relatively small tightly trapped condensates [11, 22], although this may change with the development of trapping technologies. If the target condensate does not satisfy these requirements, more complex measurement and feedback is required. For moderately sized condensates a cavity-mediated measurement with an extra control appears promising. The shape of the quantum noise control suggested in Eq. (5) must match the measurement, and it might be possible to implement this feedback using the probe beam itself. For very large loosely trapped condensates, it is likely phase-contrast imaging will be the only option, in which case full spatial control of the trapping potential will be required.

This letter has investigated the previously unexplored full quantum-field dynamics of a feedback-cooled multi-mode BEC undergoing (a) a cavity-mediated continuous cosine measurement, and (b) direct continuous density measurement via phase-contrast imaging. Full quantum-field simulations, performed with the recently developed NPW method, have revealed regimes with additional measurement-induced heating that is *not* included in single-mode and semiclassical models, and furthermore cannot be counteracted with simple linear and quadratic

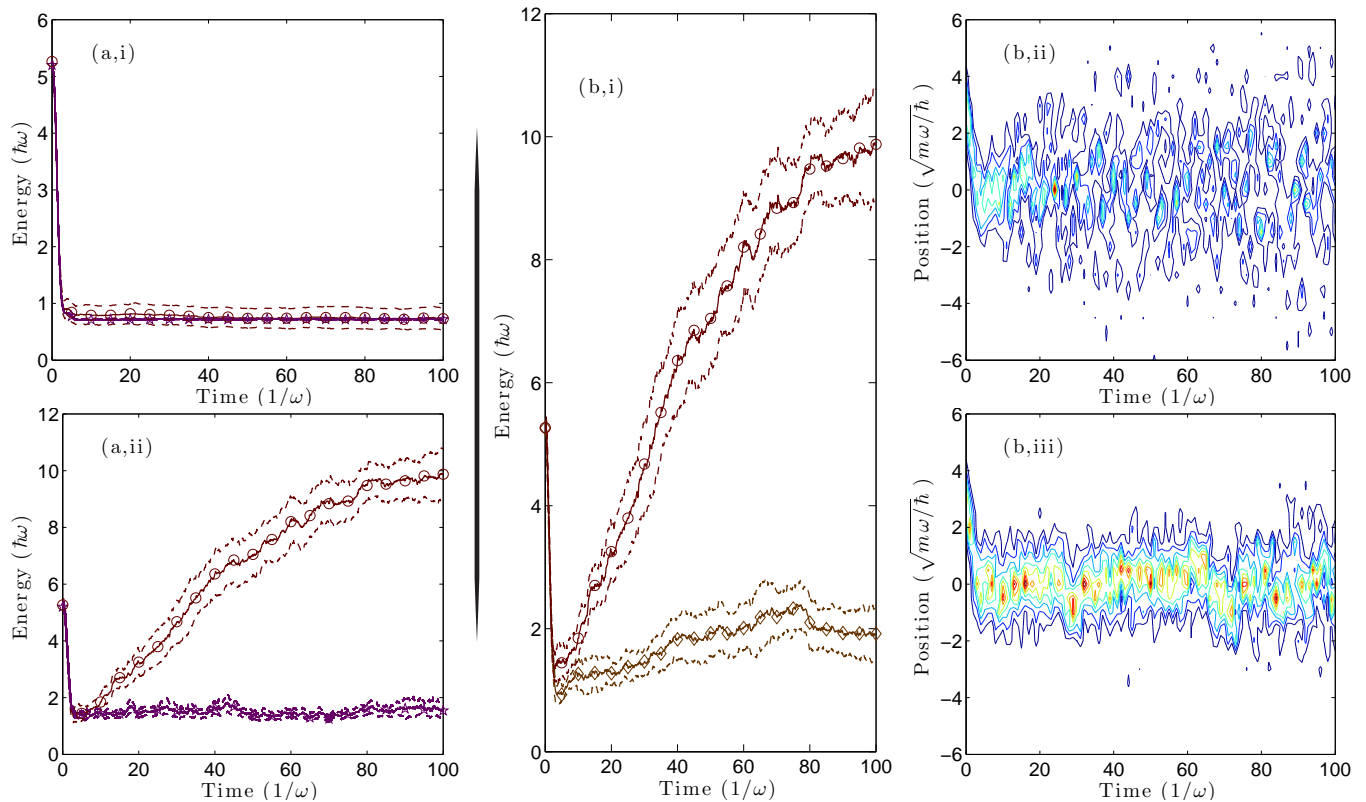


FIG. 3: (a) Comparison of the Hartree-Fock method versus NPW for simulation of a cooled BEC under direct measurement for different ν strengths. The NPW representation was integrated using Eq. (4), averaged over 10 paths, and plotted in red with circles. The Hartree-Fock representation was integrated using Eq. (2), averaged over 100 paths, and plotted in purple with stars. A coarse resolution measurement, $\nu = 10$, is plotted in (a,i) while fine resolution measurement, $\nu = 0.1$, is plotted in (a,ii). Both plots have $\gamma = 1$, $N = 100$, $U/N = 3$ and $u_s = 1$. The Hartree-Fock method only agrees with NPW when $\nu = 10$, it fails to predict the additional heating present in the system when $\nu = 0.1$ (b) A comparison of a controlled BEC under direct measurement without and with the quantum noise control averaged over 10 paths. (b,i) shows a comparison of the per particle energy of the system, while the density fluctuations in the system without and with control are shown in (b,ii) and (b,iii) respectively. In (b,i): integration of the NPW representation Eq. (8) without quantum noise control, $u_\mu = 0$, is plotted with red with circles; integration with the addition of the quantum noise control, $u_\mu = 5$, is plotted in orange with diamonds. (b,ii) and (b,iii) are contour plots of the BEC's density, all lines indicate equidensity parts of the BEC, the colour range of the lines range from (light) blue hues to (dark) red hues to indicate low density to high density respectively. All integrations were performed with $u_s = 1$, $\nu = 0.1$, $U/N = 3$, $\gamma = 1$ and $N = 100$.

feedback controls. Fortunately, we have developed more sophisticated quantum-noise controls for both measurement schemes that cancel this additional heating, and shown that this allows for successful feedback-cooling of the condensate. This has been an important demonstration of the necessity of full quantum-field simulations, and in particular the NPW method, to the successful design and implementation of measurement-based feedback-control schemes for multi-mode quantum systems.

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